On a Multi-Commodity Queueing Inventory System with Environment-Dependent Arrival and Service Rates

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Abstract

Demand for products can be influenced by various environmental factors, such as economic conditions, seasons, technological changes, and consumer preferences. In an environment-dependent demand system, the demand for each product varies according to the prevailing environment. For instance, certain products may experience higher demand in specific seasons or during particular economic conditions. While all items have their own demand in all environments, specific items experience significantly higher demand in specific environment. This specific demand pattern requires tailored inventory management strategies for each commodity-environment pair, ensuring that the system can dynamically adapt to changes and maintain optimal stock levels.

In this model, there are N environments and N commodities, where each commodity is highly demanded in its respective environment. In the r^{th} environment, customer arrivals follow a Poisson process with rate λ_r, for $r = 1, 2, \ldots, N$. Upon the arrival of a customer, if the server is idle and the requested item is available, the customer is served immediately and leaves the system forever. If not, the customer waits in an infinite-capacity queue with a first-come-first-served policy until their turn arrives and the item becomes available. The service time of the l^{th} item in the r^{th} environment is exponential with parameter μ_r^l for $r, l = 1, 2, \ldots, N$, where $l \neq r$. The service time of the r^{th} item in the r^{th} environment is i.i.d. with a generalized Erlang distribution of order k. The inventory is replenished according to the (s_l, S) policy, where s_l depends on the inventory type. The lead time for replenishment of the l^{th} item in the r^{th} environment is exponential with parameter β_r^l for $r, l = 1, 2, \ldots, N$. The environments change cyclically with an exponential distribution parameter α. The system is studied in detail; the steady-state probabilities of the system states are computed, and some performance measures are derived. A cost function is constructed using these performance measures, and the optimal values of the pair (s_l, S) are studied numerically. Additionally, the system is examined by introducing a local

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purchase of one unit with negligible service time, and the performance is compared with the original model.

This systems has broad applications across various industries. Retailers can manage diverse product categories more effectively by considering seasonal trends, economic conditions, and consumer preferences. This approach leads to improved demand forecasting, optimized inventory levels, and enhanced customer satisfaction. In the healthcare sector, it is crucial to maintain optimal levels of medical supplies and pharmaceuticals. Demand for these items can fluctuate due to factors such as disease outbreaks, seasonal illnesses, and advancements in medical treatments. In communication networks, bandwidth requirements for different applications (e.g., video streaming, VoIP, file transfers) can vary based on environmental factors like network congestion, time of day, and user behavior patterns.

Jacob and Krishnamoorthy [2] discuss a queueing inventory with n random environments with offer. The offer varies according to the environment. The control policy is environment dependent similar to the model discussed in this chapter. Jacob *et. al* [1] discuss a queueing inventory with one essential item and m optional items that evolves through n random environments.

Keywords: Queueing Inventory, Environment-Dependent Demand, Local Purchase.

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