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On peak age simulation in M/G/1/2 systems

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Abstract

The paper examines single-server systems with a buffer containing a single message, in which upon arrival a decision is made with some probability to admit a service request as well as two types of new requests entering the buffer: removing a message waiting for service at the time of arrival of a new one or blocking all new messages while the buffer is full. The results of modeling systems with Poisson input and lognormal distribution of service time are presented.

Keywords: *queuing system with request updates; peak age*

1. Introduction

In communication networks, the timely transmission of information is critical, as outdated data can quickly lose its actuality and relevance [1], [2]. In sensor systems about the environment the relevance of the information received is critical for tracking emergencies.

This work focuses on single-server service systems with a buffer that holds only one message at a time. We explore two strategies of service for incoming messages. First approach involves removing the current message from the buffer when a new one arrives, ensuring that the most recent data is always available. The other approach involves blocking any new messages until the existing one has been processed, prioritizing the completion of ongoing tasks.

2. Description of the system

Consider a single-server queuing system with a buffer accommodating one request [3]. In such a system there can be no more than two messages at the same time: one in service and one in the queue. The system can be in one of three states: the system is idle, one service request and the buffer is free, one service request and one waiting in the buffer.

Let T_n be the moment of arrival of the nth message, $n \geq 1, T_0 = 0$. Denote by $\tau_n := T_{n+1} - T_n$ – the intervals between the arrivals of messages to the server. For each number $n \geq 1$ introduced *admission index* of the message χ_n as follows:

$$\chi_n = \begin{cases} 1, & \text{if the message that arrived at the time of } T_n \text{ is allowed;} \\ 0, & \text{otherwise.} \end{cases}$$

Let $P(\chi_n = 1) = p$. Denote by $T_n^{'}$ the time of the message leaving the system, s_n – the time of reading the message received at the time of T_n, w_n – the waiting time for the n-th request in the buffer.

We will call the n-th message *successful* if it leaves the system after it has been fully read. Let's denote

$$\psi_n := \mathrm{I}_{T_n^{'} = T_n + w_n + s_n} \tag{1}$$

– *successful read index* The n-th message. Note that by definition, the condition $\psi_n \leq \chi_n$ is satisfied for all n.

In some applications, it may be necessary to determine the maximum age value of the information immediately before updating, or optimize the system so that the age remains below the threshold value with a certain probability. For these purposes, the *peak age* of the message is being studied. Let X_{k-1} be the time spent in the system of the previously transmitted $k-1$ message, and Y_k be the time elapsed between the completion of the service $(k-1)$-th message and the completion of the service of the k-th message. The age value reached immediately before receiving k-th update, we will call *the peak age* and define [4] as

$$A_k = X_{k-1} + Y_k. \tag{2}$$

3. Simulation results

Consider 3 types of systems. First we consider classical $M/G/1$ system without buffer. Second system is $M/G/1/2$ system with one place in buffer, where every new message after being checked for admission ($\chi_n = 1$) gets serviced if the server is idle, or gets queued if the server is busy. Moreover, new messages entering the busy system will immediately leave it without service. Thus, all messages that have arrived on the server for service or in the buffer for waiting are successful, i.e. $\psi_n = \chi_n$ for all n.

Third system we denote by M/G/1/2*. When a new message arrives in a busy system (one request is under service and one is waiting in the buffer), it replaces the one that is waiting in the buffer (the request waiting in the buffer is considered outdated). A message arriving in the system can either be immediately rejected with a probability of $1 - p$ and leave the system without being read, or it will take up space in the buffer.

Let service times s_n have lognormal distribution with density function:

$$f(x) = \frac{1}{\sqrt{2\pi}x\sigma} exp\left(\frac{-(log(x)-\mu)^2}{2\sigma^2}\right), x > 0.$$

For the considered systems, the average time spent on messages in the system and the average peak age of messages were calculated. The simulation results for $N = 2 \cdot 10^4$ and $\sigma = 0.2$, messages with different values of the parameter $\mu(\mu = 0.1, 0.2, \ldots, 3.4)$ are shown in Figures 1 and 2. The interarival times τ_n have Exponential distribution with parameter $\lambda = 2$, the probability of admitting messages into the system $p = 0.8$.

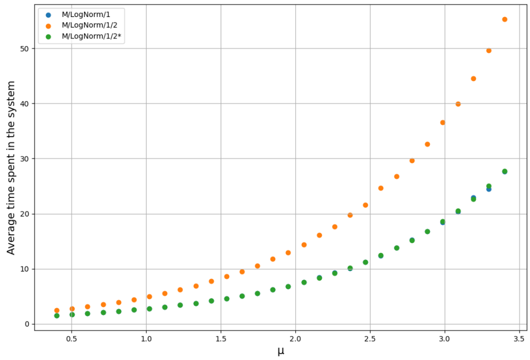

Fig. 1. The average time spent by messages in the system, depending on μ.

From Figure 1, you can see that the average time that messages spent in the system is the same for systems without a buffer and with an updating buffer. Among the considered systems, the system without a buffer shows the longest time in the system.

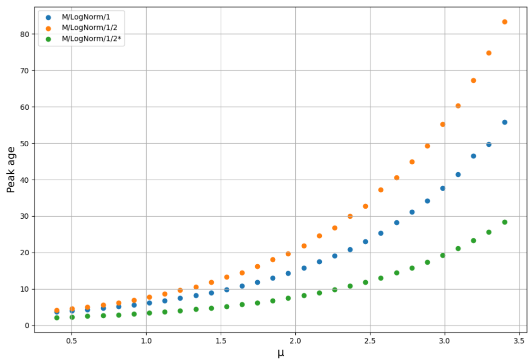

Fig. 2. The average peak age, depending on μ.

From the point of view of minimizing the average peak age, a system with an updating buffer is the best. The largest values are observed in a system without a buffer (Figure 2).

4. Conclusion

The lowest values of the peak age are shown by a system with an updating buffer. The highest peak age values are observed in a system with a buffer without updating. At the same time, in the considered cases, in terms of the average time that messages spend in the system, systems without a buffer and with an updated buffer behave in a similar way.

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