# Matrix-analytic methods in stochastic modeling

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## EXTENDED ABSTRACT

Matrix-analytic methods (MAMs) have been introduced by Neuts almost five decades ago. Since then the subject has grown significantly with applications from a variety of areas. Neuts in his famous article mentions, “..The history of the matrix methods (so called for brevity) is short, but worth telling. ..I tackled a number of models involving embedded Markov renewal processes, evidently with some measure of success, since the papers were published in noted journals and some academic recognition came my way. It privately bothered me that, as the papers grew longer and the analysis more complex, the explicit or qualitative results in them became fewer and fewer." He continues further, "..In the history of mathematics, a similarity of formalism has always indicated similarity of structure and an ultimate level of understanding is that of unifying structure."

The following quote, taken from the recently published 2-volume book of this presenter, aptly summarizes the development of MAMs. “Out of a genuine concern for the type of analysis and solutions emanating from the published works on queueing models which were driving the practitioners away from using queueing models to understand and apply them for the improvement of the processes, (late) Prof. Neuts developed phase type (PH-) distributions first in early 1970s. This helped him to lay the path for future researchers in stochastic models to study models useful in practice. These stochastic models include queues, reliability, and inventory. The impetus in developing phase type distributions resulted in Neuts to identify queues possessing matrix-geometric solutions and then to introduce a versatile class of Markovian point process (VMPP) a few years after developing the theory of PH-distributions. In 1990, this VMPP was reintroduced as a batch Markovian arrival process (BMAP) by Neuts along with his two doctoral students, David Lucantoni and Kathy Meier-Hellstern. Since then VMPP has been used as BMAP and Markovian arrival process (MAP) in the case when the arrivals occur singly. The introduction of VMPP in late 1970s paved the way for Neuts to introduce matrix-analytic methods (MAM). Ever since these methods have been extensively studied both theoretically and computationally.”

In this talk, which is based on the recently published 2-volume book on this topic to enable students and practitioners to implement the methods to study stochastic models of interest in practice, we will present an overview of matrix-analytic methods in stochastic modeling and also discuss the busy period analysis in queueing systems, in general, is very involved and complicated. Even for the simplest queueing model, M/M/1, the probability density function of the busy period is obtained in terms of modified Bessel function. A number of approaches using complex analysis, combinatorics, lattice path, and matrix-analytic methods, have been applied to study some selected queueing models in the literature. However, most of the results are in a form that is computationally challenging.

While the steady-state analysis involving queue length and waiting times of queueing models, in general, has been receiving considerable and significant attention in the literature from both analytical and algorithmic points of view, the same cannot be said (relatively speaking) about busy period analysis. This is inherent in the nature of the busy period more than by choice. In this paper, after establishing the complexity involved in the study of the busy period, we record some interesting observations on the busy period under a wide variety of scenarios through simulation approach.

The main purpose is to help researchers to look for novel theoretical and or numerical approach to solving functional and other types of equations which naturally arise in the study of busy periods, and use the simulated results here as one of the ways to confirm their results.

# Brief Bio of Srinivas R. Chakravarthy

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Srinivas Chakravarthy is Professor of Industrial Engineering & Statistics in the Departments of Industrial and Manufacturing Engineering & Mathematics at Kettering University (formerly known as GMI Engineering & Management Institute), Flint, Michigan. He was department head of Industrial and Manufacturing Engineering during 2013 – 2016.

Srinivas Chakravarthy's research interests are in the areas of algorithmic probability, queuing, reliability, inventory, and simulation. He has published more than 140 papers in leading journals and made more than 100 presentations at national and international conferences. Recently he published a 2-volume book on Introduction to Matrix-Analytic Methods in Queues – Analytical and Simulation Approach. He co-organized the First International Conference on MAMs in Stochastic Models in 1995 held in Flint. His recognitions and awards include Distinguished Faculty, 2015 (inducted into Kettering's Faculty and Alumni Honor Wall), National Science Foundation Awards, Rodes Professor, Kettering University, Kettering University Distinguished Research Award, Kettering University/GMI Alumni Outstanding Teaching Award, GMI Outstanding Research Award, and GMI Alumni Outstanding Teaching Award, and Educator of the Year Award by IEOM Society, 2016.

Srinivas Chakravarthy has significant industrial experience by consulting with GM, FORD, PCE, and UPS. His professional activities include serving as (a) Area Editor for the journal, Simulation Modelling Theory and Practice; (b) Advisory Editor for Queueing Theory and Service Management; (c) Associate Editor for the journal IAPQR TRANSACTIONS - Indian Association for Productivity, Quality & Reliability; (d) Advisory Board Member for several other journals and International Conferences; and (e) Reviewer for many professional journals.