Asymptotic analysis of M/M/K retrial queue with π-defeat under heavy load

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Retrials queueing systems (RQ) are modern mathematical models of queueing theory (QT). These models are applied to analyze and optimize various info-communication systems where repeated attempts to receive service are possible after a random interval of time. Examples of such systems are communication networks the random multiple access protocol, call centers, distributed computing centers, etc. Other important direction in queueing theory is G-systems or G-networks proposed by E. Gelenbe. These models incorporate the concept of negative influence, which is particularly relevant for info-communication systems such as DDOS-attacks, viruses, system failures, and other threats. In QT, there is a term of "negative customers", witch can have various effects: from a single processing request destroying to "catastrophes" coming.

In this paper, we consider a multiserver retrials queueing system. The arrival process of calls is Poisson with parameter λ, we will call these customers as "positive". Positive customers arrive into a service block (which has K servers), until all servers become busy. The service time of each call is distributed exponentially with parameter μ. If all servers are busy, then the call goes to an orbit where it performs a random delay. The delay duration has an exponential distribution with parameter σ. From the orbit, a call again turns to the service block. If there is a free server, the call begins its service, otherwise it returns to the orbit to make next attempt. The system also receives a Poisson process of negative call with rate γ. A negative call does not need the servicing; it brings a negative impact to the system. We consider a quite general model – π-defeat of negative calls, where an arrival negative call can "zero out" k servers with probability π_k, where $k \in 1..K$.

We denote a random process of the number of calls in the orbit by $i(t)$ and states of the service block process by $k(t)$. The problem of finding the stationary probability distribution of the number of calls in the orbit is posed. Process $\{k(t), i(t)\}$ is Markovian, thus we compose a system of Kolmogorov equations for probability distribution $P(k, i, t)$. We propose the method of asymptotic analysis for the system of equations solving under a heavy load condition.

We denote the system load by $\rho = \lambda/K\mu$. So, the system operates in stationary mode, when $\rho < S$, where S – system throughput. We solve the system of equations under heavy load condition $\rho \to S$. The formula for the system throughput S is derived in the paper. We proved that asymptotic characteristic function of the number of call in the orbit under heavy load condition have a gamma-distribution form with finding parameters.