Partial asymptotic analysis method for two-class retrial queues with constant retrial rate *

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Modern telecommunication networks have several types of information. It can be text, sound, image, service information, etc. Transmission and processing of heterogeneous data need more complex preliminary analysis. One of the example of heterogeneous networks is multimodal systems. Most of studies in multimodal system are based on simulation modelling. Although, various mathematical models of such systems have been proposed, their analytical analysis is usually not carried out due to the need to study multidimensional random processes.

We consider a retrial queueing system with two classes of customers. A customer of the n-th class comes to the system according Poisson arrival process with parameter λ_n, where $n = 1, 2$. There is one server. If the server is idle, the n-th class customer starts its servicing during the exponentially distributed random time with rate μ_n. If an arrival customer finds the server busy, it goes to the orbit and makes a random delay distributed exponentially with rate σ_n. We suppose that that the constant retrial policy takes place, so the only first customers in each orbits can access to the server.

We denote a random processes of the number of the n-th class calls in the orbit by $i_n(t)$, where $n = 1, 2$. Process $k(t)$ determines states of the server, so $P\{k(t) = k, i_1(t) = i_1, i_2(t) = i_2\} = P(k, i_1, i_2, t)$. We write a system of Kolmogorov equations for stationary probability distribution $P(k, i_1, i_2)$. From the system, we find the stationary probabilities of the server states R_k and the stationary probabilities of the state of the idle server and an empty one orbit. Due to non-negative values of probabilities, we also derive the conditions of the orbits stability (partial too).

For obtaining the probability distribution, in the paper we propose the method of partial asymptotic analysis of the one class of customers under the condition of a heavy load of the other class of customers $\lambda_2 \to S_2$ (when $\lambda_1 < S_1$), where S_n is maximum possible values of n-th arrival process rate when the n-th orbit is stable.

In this way, we have obtained the formula for asymptotic characteristic functions of number of customers of other class, from which we can find marginal characteristics of the considered model. Asymptotic formulas will give us more precise results if the rate of one of the arrival processes tends to its throughput: $\lambda_k \to S_n$.

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