Revisiting the transient analysis of the two-stage tandem network with a sharing finite capacity buffer

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Abstract. Consider the tandem network (queue) consisting of two stages with one server at each stage, serving customers one-by-one in an FCFS fashion. The service times at stage $i \in \{1, 2\}$ are i.i.d. random variables and have the exponential distribution with the parameter μ_i. A single Poisson flow of customers arrives at rate λ to the 1^{st} stage, and after completing the service at the 1^{st} stage it has to go through the service at the 2^{nd} stage. It is assumed that the total capacity of the network is finite, say N, i.e. there is a buffer of size $N-2$, in which customers await service. A newly arriving customer gets accepted if the total number of customers in the network is less than N. We are interested in the computation of the basic transient performance measures of such networks (e.g. time-dependent loss probability).

Tandem queues (and this specific queue as well) are well-known in the operations research community (e.g. [1,2,3]) and find its applications in various fields (e.g. [4,5]). Due to the very simple assumption considered, most of the answers (and the transient distributions as well) can be obtained by adopting the available results for the QBD processes (i.e. matrix analytic methods).

If $p_{ij}(t)$ is the probability that at instant t the number of customers at the 1^{st} stage is i and at the 2^{nd} stage is j, then the set $\{p_{ij}(t), 0 \le i+j \le N\}$ satisfies (in terms of the Laplace transform (LT)) the finite system of linear algebraic equations. We try to get some insight into the interdependence between the LT of $p_{ij}(t)$ by analysing this system using the generating functions.

We will show that the use of this conventional method allows one to compute the LT of some performance measures (e.g. LT of the loss probability) from the system of N equations (compared to $(N^2 + 3N + 2)/2$ equations required by straightforward methods). From this solution the LT of the whole distribution $\{p_{ij}(t), 0 \le i + j \le N\}$ can be computed recursively (although not very efficiently). The inverse transform must (and can) be performed numerically. As a side result we show that, using the symmetry of the transition diagram under the cyclic permutation of the transition rates, the LT of the boundary probabilities can be computed from $3N$ equations. Finally, since the obtained results are valid for any initial distribution $\{p_{ij}(0), 0 \le i + j \le N\}$, it allows one to build simple approximations for transient performance measures in the case of periodic intensities.

Keywords: tandem queues · finite capacity · recursive solution · random walks

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