Performance Analysis of $GI|GI|n,m$ queues using Marked Markov Processes*

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Abstract. We are investigating the main performance characteristics of a finite-buffer multi-server queuing system with arbitrarily distributed inter-arrival and service times. Following a slightly modified Kendall's notation for queuing systems, this type of system is denoted as $GI|GI|n,m$. To analyze this system, we are utilizing a new concept called as Marked Markov Processes (MMP). By following the suggested approach, a mathematical model of the system is constructed, marks' transformations and analytical expressions for calculating their distributions are given. The procedure for calculating the model's probabilistic and temporal characteristics relies on a simulation algorithm directly derived from the theoretical findings. Numerical study involves validation of the proposed method through comparison with well-known $M|M|1$ and $M|GI|1$ models. Additionally, the sensitivity analysis of the main model's performance measures to some input parameters, including inter-arrival and service time distributions, corresponding coefficients of variation, traffic intensity, is also studied.

Keywords: $GI|GI|n,m$ queuing system, Marked Markov Processes, arbitrary inter-arrival and service times distributions, steady-state probabilities, mean number of customers, mean number of busy servers, mean sojourn time, mean response time, sensitivity analysis.

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