On a Single-Commodity Queueing Inventory System with varying demand quantities

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Abstract

We consider a single-server, single-commodity queueing inventory system managed under the (s, S) inventory control policy. The system is characterized by two distinct types of customer demands, where Type1 customers request a single unit of the item,and Type2 customers request multiple units less than s.The arrival of customers follow independent Poisson processes, with rates λ_1 for Type1 customers and λ_2 for Type2 customers. The service times are exponentially distributed with distinct service rates μ_1 and μ_2 corresponding to the two customer types.

The inventory management follows the (s, S) policy, wherein orders to replenish inventory are placed when the stock level falls below s, and the maximum storage capacity is S. The replenishment lead time is assumed to be exponentially distributed with a rate β. We impose the constraints: the number of Type2 customers in the system can not exceed a fixed integer N, and Type2 customers are only served if there are no Type1 customers in the system and the inventory level is at least 2. Also,when the demand quantity is more than the stock level, a Type2 customer will be served with all items left in stock ensuring of at least 2 units, and when the inventory is out of stock all the Type2 customers will leave the system.

We employ a Continuous Time Markov Chain (CTMC) to represent the state of the system, where the state space is defined by the number of Type1 customers, Type2 customers, and inventory level at any given time. Key transitions within the system - due to customer arrivals, service completions, and inventory replenishments- are modeled, and the infinitesimal generator matrix Q is constructed accordingly and the system stability condition is obtained. Some important system performance measures are derived.

In retail industry , retailers often deal with varying customer demands - some purchase single items (like Type1 customers), while others buy in bulk (similar to Type2 customers). In health care sector, for example a hospital may face routine demands for single doses of medication (Type1) or bulk requests during emergencies or surgeries (Type2). E-commerce companies and warehouses face the challenge of fulfilling orders of various sizes efficiently. Incorporating varying demand quantities,the model provides valuable insights in to the dynamics of inventory control.